

CONCEPT NOTE VOL. 4, JANUARY 2014

# Scientific Computing and Development Economics

This CFSP Concept Note addresses the need for advanced computation in the field of development economics. While computational modeling is a critical part of research in the natural sciences and engineering field, it is even better-suited for use in development economics, a field which studies the highly complex structures of countries in transition. Computational models provide researchers with a precise and cost-effective tool to observe and interpret patterns in observed data and to understand these within the context of economic theory.

#### NEED FOR ADVANCED COMPUTING

Computational modeling is now a critical and essential part of both academic and applied research in the natural sciences and engineering. It has replaced many costly and often impossible experiments with precise, trustworthy results made available by relatively cheap high performance computing. Advances in computing also enable researchers to explore solutions for models of higher complexity where neither analytical nor experimental methods can be directly applied.

Perhaps surprisingly, the need for advanced computing is even greater in development economics. Though people typically associate development with poverty and less advanced technologies of production in emerging economies, this does not also mean that the primary research methods should be limited to field experiments and data collection. In fact, it is precisely

because these economies are typically economies in transition, with many frictions clearly present in the data, that the need for numerical modeling is greater.

Developed economies don't typically face such a variety of frictions, and often, many features of these economies can be described by highly stylized tractable (toy) models with intuitive and simple analytical solutions. Experiments to study particular policy effects leading to off-equilibrium transitions in developed economies are extremely costly and difficult to implement, as the inertia of global market prevents the effects of local policy from being clearly observable. At the same time, regional features in developed countries vary much more, and local effects can be separated from aggregate shocks.

Thus, development economics can be thought of as both the testing stage where the most fundamental principles of economics can still be observed in action and a very rich natural environment where a multitude of economic models with highly complex structures need to be computationally tested against observed data to provide a spectrum of data-driven ideas about ongoing change.

The nature of frictions and the highly diverse environment typical of development economies precludes in most cases analytical simplifications and demands highly complex computations to be performed, not only for direct data analysis but also mostly to provide a basic intuition, a lens, through which the patterns of processes observed sharpen and become open to interpretation.

#### KEY ASPECTS OF ECONOMIC ANALYSIS

Complex economic models require the application of numerical analysis to choose the best or most feasible way of approximating and finding a solution by numerical methods and, subsequently, to determine the best or most feasible computational technique for the implementation of algorithms on the computing platform.

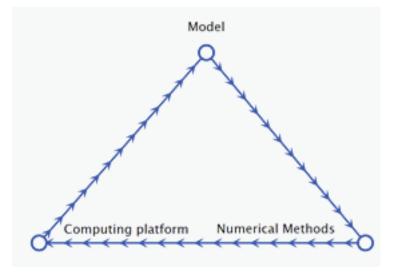
# A model, numerical methods, and a computing platform

As shown in Figure 1, one can think of economic analysis as consisting of three key parts: a model, the numerical methods, and the computing platform.

All three parts are thus inherently linked with the ongoing process of improvement in the accuracy of numerical results and the feasibility of the solution. This process of improvement adds more complex and interesting classes of models to the researcher's toolbox.

#### FIGURE 1

Key parts of economic analysis



### Factors impacting the complexity of models

What increases the complexity of economic models? In particular, the following aspects of applied general equilibrium development models studied by the Consortium's researchers require advanced computation:

- Global and regional inequalities;
- Incomplete markets and partial risk-sharing, with multiple equilibria possible;
- Scenarios in which choices are endogenous (i.e., derived internally), rather than obtained via exogenous restrictions (i.e., derived externally) over the range of possibilities;
- Diverse occupational choices of individuals, on both the supply and demand side;
- Shifting preferences on both the demand and supply side;
- Risk taking, particularly with non-unique values and within policy functions;
- Capital market structure and the associated dynamics;
- Heterogeneity among individuals rather than a single representative agent; and
- Spatial dimensions with transportation costs and spatially segregated markets.

## Example: A model of entrepreneurship

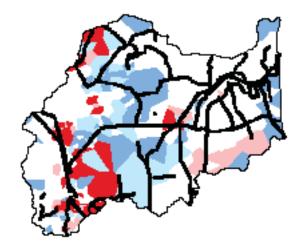
As an example of how economic models are used in practice, we look at the Lloyd-Ellis and Bernhardt (LEB) general equilibrium model of entrepreneurship with a financial sector. This model allows researchers to look at actual data in developing countries through a lens of an "alternative timeline" – i.e., what would happen to the country if a specific policy was changed.

In one exercise, we freeze the credit conditions at a point in time to see how important economic growth is for that particular sector of the economy. Naturally, we need first to calibrate all of the parameters of the economy carefully to match the actual data and leave the remainder of the production factors intact over the time while we constrain only the financial sector.

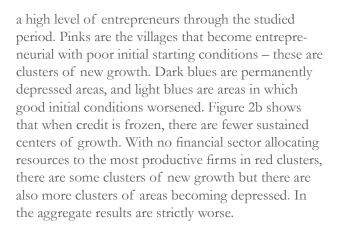
Figure 2 illustrates the outcomes of this exercise, with a comparison of actual data for the Lop Buri province (2a) and the model prediction with credit frozen for Lop Buri (2b). Areas in red are villages initially with a higher-than-average concentration of entrepreneurs that sustain

#### FIGURE 2

Lop Buri, 1986 - 1996



Actual data

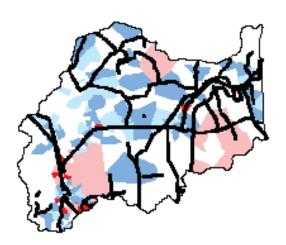


Although simplistic and far from being a comprehensive model of the financial sector, the LEB model produces remarkably strong evidence for the critical importance of finance for development. This finding is in accordance with empirical data for Thailand but also with impressible counterfactuals that clearly stress the contribution of finance to growth.

#### ADVANCED SCIENTIFIC COMPUTING:

#### **VECTORIZATION AND PARALLELIZATION**

One way in which advanced computing can be applied is through vectorization and parallelization, or use of a parallel computing structure. This architecture allows a computer to process one operation on multiple processors simultaneously. Vectorization and parallelization often improve efficiencies and increase



Model prediction with credit frozen

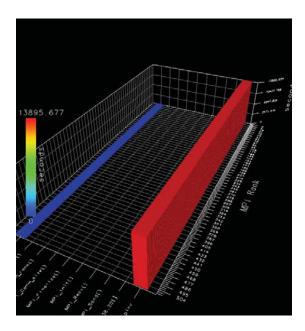
performance, particularly when working with large datasets, though there are some limitations, as discussed here.

Many economic problems can be seamlessly transferred to computing clusters with ordinary parallelization utilizing a Message Passing Interface (MPI). For example, small province-based parallel estimations on a national scale can be used to study the regional and local effects of a policy. Parallel structural estimation with scans of high-dimensional specification parameters are also both essential for robustness testing and relatively easy to implement. Figure 3 shows the computational time gain by using parallel computing on the IBM Blue Gene supercomputer efficiently, in which computing time decreased from 2 months to under 4 hours without any losses due to communication overhead between the nodes.

There is also a larger and more general issue of how to use the modern computation architecture efficiently for economic modeling since all modern CPUs are, in a sense, parallel processors. Indeed, there is a so-called "Ninja gap", or the performance gap between naively written computer code that is unaware of parallelism and the best-optimized code on modern multi-/many-core processors. That gap ranges from one order of magnitude to two and

#### FIGURE 3

#### Time savings in computing



more orders of magnitude in terms of computation cost, and it can make a difference between a problem being completely unsolvable and trivially solvable.

In this respect, computational complexity and the complexity of numerical algorithms are not necessarily complementary. It can be optimal to substitute a highly complex numerical algorithm with a simpler one by using higher complexity in computational implementation.

Modern massively parallel processors strongly favor simpler numerical algorithms with larger block-independent data frames optimally processed with vectorized function calls versus sophisticated (and expensive) serial code based on excessively complex numerical analysis algorithms.

As a result, advanced computational skills – particularly the ability to efficiently use modern massively parallel computing architectures – are now at a premium as compared with past trends in advances in pure applied mathematical techniques.

#### TOOLS FOR ADVANCE SCIENTIFIC

#### COMPUTING: OPTIMIZATION

The basis for solving most economic models is some kind of optimization problem, either in a deterministic (non-random) or stochastic (random) environment with different degrees of heterogeneity and either linear or non-linear constraints. Here, we focus on comparing solutions for two types of models: commonly used convex models -- in which the predicted economic behavior is well characterized and understood -- and more complex non-convex models -- in which the properties of frictionless competitive markets may not hold.

Most of the models for developed countries, as well as some simple models for developing countries, allow researchers to assume convexity in a strong or weak form for both the objective function and constraints. Such models have a well-defined and often easily reachable unique solution. Given this model structure, the usage of fast Newton-based non-linear solvers with user-defined Jacobians and Hessians makes most sense.

Advanced "black-box" solvers such as the commercially-available solver KNITRO or the open source solver IPOPT are comparable in quality and in quickly finding a global optimal solution for convex problems without too much effort from the user. The research problems can be quickly and transparently coded in specialized languages such as GAMS or AMPL or in any of the scientific programming languages such as Fortran, Python or MATLAB.

However, more complex economic models, especially ones with moral hazard and adverse selection, are non-convex. Finding the true global optimum is often not feasible, even with multi-start option in non-linear solvers. Solving such models is still often an art rather than science since even existence of a solution is not guaranteed. Although there are algorithms that sometimes allow researchers to find an approximate solution, there is always a possibility that it is only a local optimum.

## A solution for more complex economic models

Prescott and Townsend (1984) develop a lottery technique to overcome challenges in finding the optimal solution to a research question. In a lottery formulation, any non-linear non-convex problem can be written as a very large constrained linear program which is guaran-

teed to have a global solution if it is feasible. Prescott and Townsend's work specifically targets the problem of non-optimality, nonexistence, and multiplicity in both theoretically and empirically important cases of adverse selection and moral hazard in labor force and insurance. The contract lottery technique is broadly applicable to most of non-convex problems in economics. The size and non-trivial properties of those linear problems require the usage of the most advanced linear solvers, such as GUROBI.

### Choosing a linear programming solver

The percentage of problems solved in a given benchmark set of LP problems by the open source solver lp\_solve is approximately 6%. At the same time, the commercial solver GUROBI which is free for academic use, solves approximately 90% of test problems at around 20 times the speed.

The choice of the linear programming solver matters both in terms of the computational cost of solving the problem and, more importantly, in the accuracy of solution obtained. With high-dimensional problems to be solved in cases of moral hazard and adverse selection on dense grids, it is essential to control for potential numerical instabilities caused by the inability of a particular solver to find the right solution.

# OPTIMIZATION WITH WEAKENED ASSUMPTIONS

Economics was at the frontier of digital computations and numerical optimization in the early 1950s, with some of the major advances in algorithms having their origin in important economic problems being solved at the time.

For example, Arrow & Solow originally introduced the well-known Augmented Lagrangian set of optimization methods in 1958. They identified a constructive algorithm to find general equilibrium solution for problems with non-concave objective functions based on economic intuition. In particular, their insight about using expected price movement rather than existing price level to update the search direction for equilibrium is both a sound economic and valuable numerical technique to allow for faster convergence.

Later, in the optimization literature, those findings

became widely known as methods of multipliers (Hestenes,1969, and Powell, 1969). Augmented Lagrangian methods are especially useful in Industrial Organization research that joins mechanism design and game theory to find the optimal competitive market outcome in off-equilibrium cases applicable to developing countries in transition (Moll, Townsend & Zhorin, 2013). More generally, there are other classes of non-convex general equilibrium models with transitions that can benefit from applying ideas developed in pioneering work by Arrow & Solow.

Finally, HOPSPACK (Hybrid Optimization Parallel Search PACKage, developed at Sandia National Laboratory) has an asynchronous pattern search solver (supports MPI, OpenMP) over a user-defined objective and nonlinear constraint functions (that can be expressed in Fortran, MATLAB, and Python). HOPSPACK can also be efficiently applied to economic models where no analytical or numerical Jacobians and Hessians can be obtained.

# A HYBRID APPROACH TO COMPLEX ECONOMIC PROBLEMS

Based on the experience of CFSP's researchers, the following hybrid approach is recommended when dealing with complex, non-convex informational problems in economics when no priors are readily available to establish the properties of an optimal solution.

Step 1: Solve the linear programming problem with lotteries on coarse grids with a guaranteed solution which can serve as a good starting point in the vicinity of the global optimum.

Step 2: Use this information to exclude bad (i.e., nonsensical) local traps from the non-linear constrained optimization.

Step 3: (locally convergent only!) Combine prior steps with a multi-start option in an advanced non-linear solver that allows a researcher to converge quickly on what is – hopefully -- a true global optimum.

Step 4: (optional) Iterate or facilitate structural estimation of model parameters on real data samples.

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# Concept Note

#### ABOUT

CFSP Cocnept Notes are designed to support policy-makers and researchers by providing reputable summaries of key literature on select topics in the field of development economics.

The Consortium on Financial Systems & Poverty is a research organization of leading and up-and-coming economists. Our goal is to alleviate global poverty through helping to identify, define, and develop more efficient financial systems. We strive to generate tangible and objective results that have meaningful lessons for policymakers, researchers, and stakeholders. We are based at the University of Chicago and led by faculty director Robert M. Townsend of MIT.

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